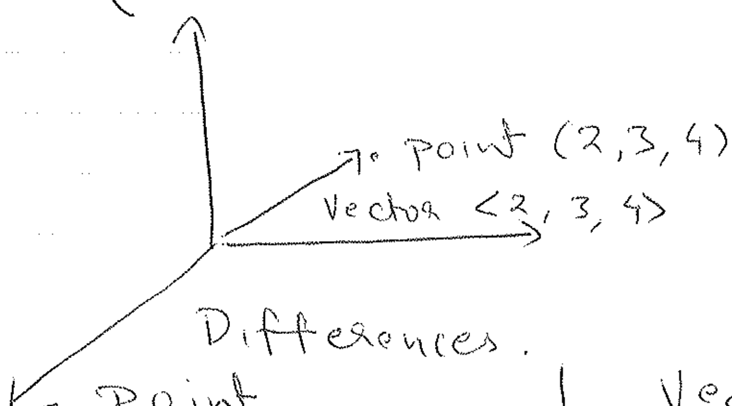
(1)

Course

- Several Variables (3-weeks)
- Integration (5 weeks)
- Probability (1 week)
- Sequences and Series (3 weeks)

1st week: Surfaces.

- Planes (René Descartes precursor to Newton and Leibniz)
- Vectors (William Hamiltonian)



Differences.

Point	Vector
Curvy bracket	Pointed bracket.
without arrow	with arrow.
Represent a coordinate.	Represents magnitude and direction.
Dependent on origin (e.g. go to 47N, -47E).	not dependent of origin (go 200 km East)

Say

Vector from point (p_1, p_2, p_3) to (q_1, q_2, q_3) is $\langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle$

Vector addition: $\vec{u} = \langle 2, 3, 4 \rangle$
$\vec{v} = \langle -3, -6, 2 \rangle$

$$\vec{u} + \vec{v} = \langle 2 + (-3), 3 + (-6), 4 + 2 \rangle$$

$$= \langle -1, -3, 6 \rangle$$

$$2\vec{u} = \langle 2.2, 2.3, 2.4 \rangle$$
$$= \langle 4, 6, 8 \rangle$$
$$\cancel{2\vec{u}}\ 3\vec{v} = \langle 3.(-3), 3(-6), 3(2) \rangle$$
$$= \langle -9, -18, 6 \rangle$$
$$2\vec{u} - 3\vec{v} = \langle 4 - (-9), 6 - (-18), 8 - 6 \rangle$$
$$= \langle 13, 24, 2 \rangle$$

Magnitude: $\vec{v} = \langle v_1, v_2, v_3 \rangle$

The magnitude is $\sqrt{v_1^2 + v_2^2 + v_3^2} = |\vec{v}|$

Direction: $\hat{v} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{1}{\sqrt{v_1^2 + v_2^2 + v_3^2}} \langle v_1, v_2, v_3 \rangle$

Dot product: $\vec{u} = \langle u_1, u_2, u_3 \rangle$
$\vec{v} = \langle v_1, v_2, v_3 \rangle$

The dot product is $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

Orthogonal / Perpendicular vectors

$\vec{u}$ and $\vec{v}$ are orthogonal / perpendicular if

$\vec{u} \cdot \vec{v} = 0$

Parallel vectors

$\vec{u}$ and $\vec{v}$ are parallel if

$$\hat{u} = \hat{v} \quad \text{or} \quad \hat{u} = -\hat{v}.$$

Example 1)

$\vec{u} = \langle 0, 3, 4 \rangle$ $\qquad\qquad$ $\vec{v} = \langle 3, 4, 0 \rangle$

1) $|\vec{u}| = \sqrt{0^2+3^2+4^2} = \sqrt{0+9+16} = 5.$ $\qquad$ $|\vec{v}| = \sqrt{3^2+4^2+0^2} = \sqrt{9+16+0} = 5$

2) $\hat{u} = \dfrac{\langle 0, 3, 4 \rangle}{5} = \langle 0, \frac{3}{5}, \frac{4}{5} \rangle$ $\qquad$ $\hat{v} = \dfrac{\langle 3, 4, 0 \rangle}{5} = \langle \frac{3}{5}, \frac{4}{5}, 0 \rangle.$

3) ~~$\hat{u} \cdot \hat{v}$~~ Dot product $\vec{u} \cdot \vec{v} = \langle 0, 3, 4 \rangle \cdot \langle 3, 4, 0 \rangle = 12.$

- $\hat{u} \neq \hat{v}$ and $\hat{u} \neq -\hat{v}$ so $\vec{u}$ is not parallel to $\vec{v}$.
- $\vec{u} \cdot \vec{v} \neq 0$ so $\vec{u}$ is not orthogonal to $\vec{v}$.

Planes

A plane is a set of points (x, y, z) such that $\underline{ax + by + cz = d}$ for some a, b, c, d.

$ax + by + cz = d$ is the equation of the plane.

How to check if a point lie on a plane?

Does $(2, 3, 4)$ lie on $2x + 3y + 5z = 10$

Substitute $(2, 3, 4)$ in place of (x, y, z)

$$2(2) + 3(3) + 5(4) = 33 \neq 10.$$

$(2, 3, 4)$ does not lies on $2x + 3y + 5z = 10$

Normal to the plane — Normal vector to $ax + by + cz = d$ is $\langle a, b, c \rangle$

Two planes are parallel if their normal vectors are parallel.

Two planes are perpendicular if their normal vectors are perpendicular.

<u>Example2)</u> Are $3y+4z=6$ and $3x+4y+\cancel{z}=10$ parallel or perpendicular?

$3y+4z=6$

$\Rightarrow 0.x+3y+4z=6$

normal vector is
$\langle 0,3,4\rangle$

$3x+4y+0.z=10$

$\cancel{3x+4y}$

normal vector is
$\langle 3,4,0\rangle$

In Example 1 we proved $\langle 0,3,4\rangle$ and $\langle 3,4,0\rangle$ are ~~neither~~ ~~not~~ neither parallel nor orthogonal.

<u>Example(3)</u> — <u>Very important</u>

Find plane passing through $(0,2,1)$ and parallel to $2x+y+z=15$.

- Normal vector of $2x+y+z=15$ is $\langle 2,1,1\rangle$

Equation is of form $2x+y+z=a$.

Need to find 'a'

Plug in point $(0,2,1)$ in the equation.

$$2(0) + (2) + (1) = a$$

$$\Rightarrow a = 3$$

The equation ~~is~~ is

$$\cancel{8x + 9y + 10z = 1}$$

$$2x + y + z = 3$$

Steps: 1> Find normal.

2> Write the form of equation.

3> Plug in the point

4> Find 'a'

5> Write down the equation.