Traces and level Curves

9th January.

Surfaces $\quad 10x + 3y + 5z = 2$ (Plane)

$10x^2 + 3y + 5z = 2$ (In general)

$\boxed{\text{How to understand a surface ?}}$

Traces. $\qquad\qquad$ level curves

Traces: Intersection of a plane parallel to one of the coordinate plane and a surface

xy trace : Intersection of surface and $z=0$

yz trace : Intersection of surface and $x=0$

xz trace: Intersection of surface and $y=0$.

In general $x=a$, $y=a$ or $z=a$ for some ~~no~~ number a.

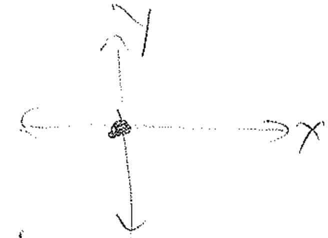
- Draw traces of $x^2+y^2-z=0$.

(1) $z=a$. $\quad x^2+y^2=a$

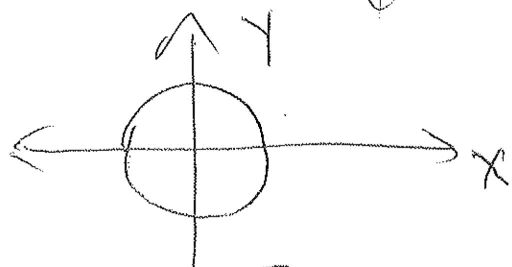
Sum of squares, $a>0$

2

$x^2+y^2=0 \quad (a=0)$



$x^2+y^2=1 \quad (a=1)$



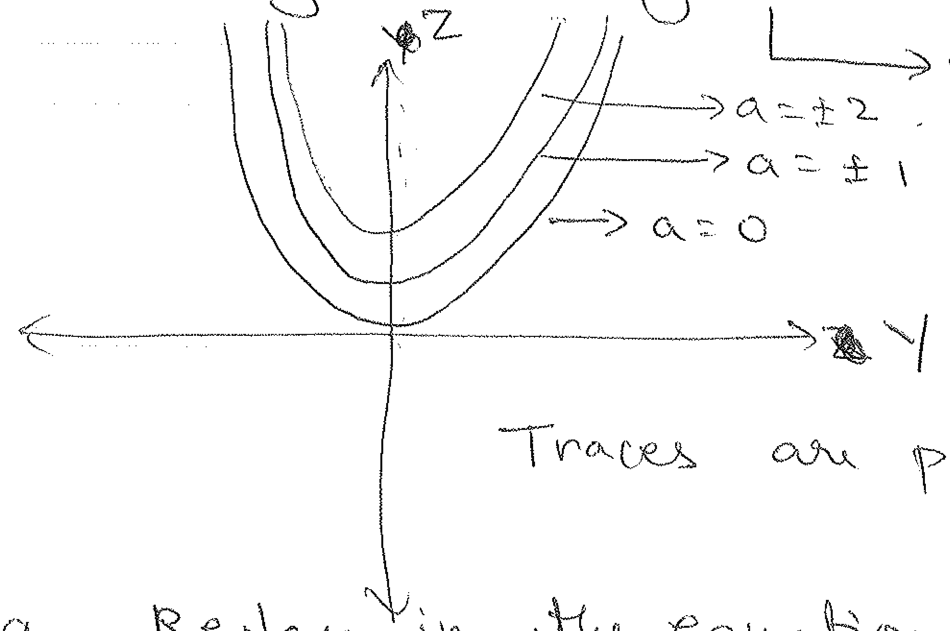
Traces are circles

(2) $x=a$

Replace in the equation.

$a^2+y^2-z=0$

$\Rightarrow \cancel{y^2 \quad z = a^2} \quad y^2 = z-a^2$ → parabolas.

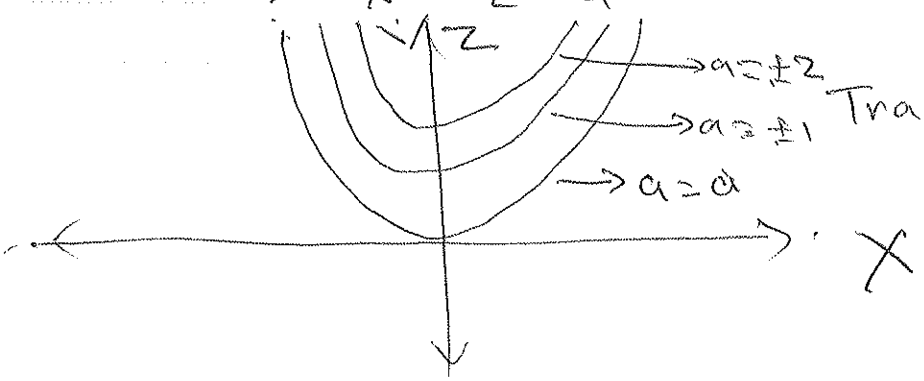


Traces are parabolas

(3) $y=a$, Replace in the equation.

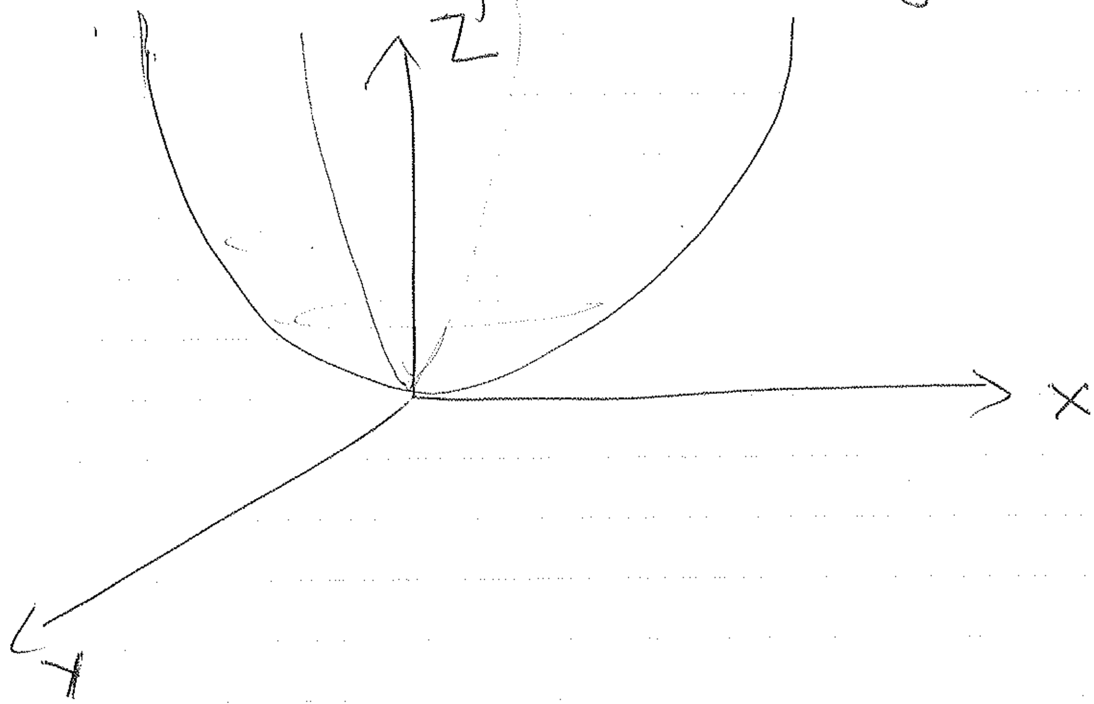
$x^2+a^2-z=0$

$\Rightarrow x^2 = z-a^2$.



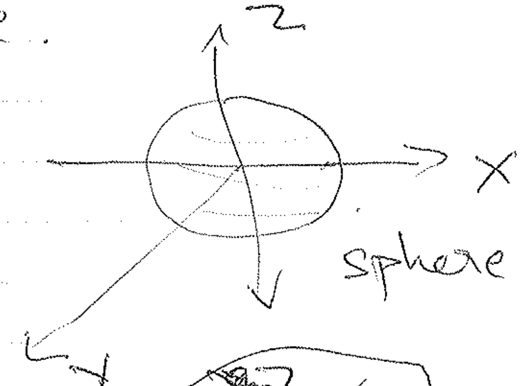
Traces are parabolas.

3 dimensional surface. $x^2+y^2-z=0$

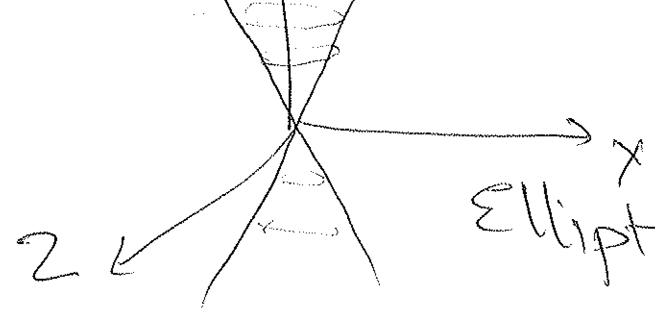


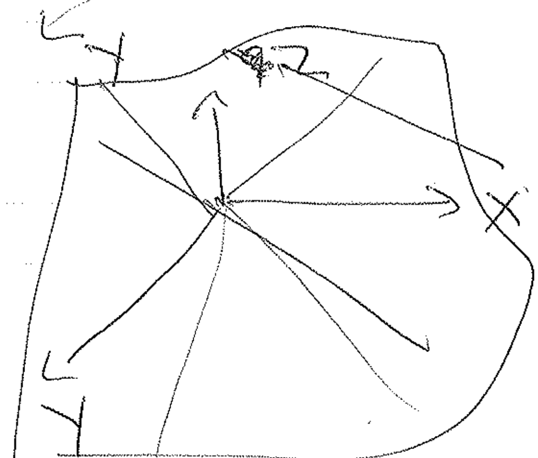
Exercises: ~~x^2+y^2+z~~ Draw the traces and guess the shape.

1) $x^2+y^2+z^2=1$



2) $z^2=\frac{x^2}{b^2}+\frac{y^2}{c^2}$





Elliptic cone.

~~Multiv~~ Functions of two variables

Examples:

1) $f(x,y) = x^2+y^2$ — Also written as $z = x^2+y^2$.

2) $f(x,y) = \ln xy$

3) $f(x,y) = \sin x + \sin y$.

Domain = set of (x,y) where the function is defined.

Range = set of all possible values of $f(x,y)$

For domain: • Denominator should not be 0 ($\neq 0$)

~~• log(,)~~

• log can only be taken of positive numbers (>0)

• Square root can be taken only of non-negative numbers (≥ 0)

For range: $\longrightarrow$ Remember range of general functions.

So for ~~ln~~ $f(x,y) = \ln xy$ domain is $xy > 0$

But for $f(x,y) = x^2+y^2$ domain is everything.

• Find domain and range of

(a) $f(x,y) = \dfrac{x}{x-y}$.

(b) $f(x,y) = \cos(\sqrt{x^2+y^2-2})$.

(a) $f(x,y) = \dfrac{x}{x-y}$.

<u>Domain:</u> $x - y \neq 0$ i.e. $\{(x,y)$ such that $x \neq y$.

<u>Range:</u> general technique. if

$\dfrac{x}{x-y} = z$ Choose $x = z$, $y = z-1$

So z can take any value.

$\therefore$ Range is $(-\infty, \infty)$

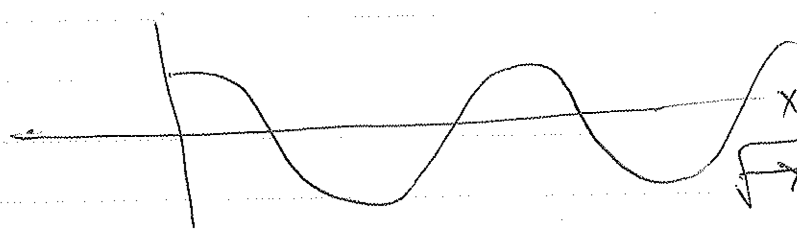
(b) $f(x,y) = \cos\left(\sqrt{x^2+y^2-2}\right)$

<u>Domain:</u> $x^2+y^2-2 \geq 0$

$\Rightarrow x^2+y^2 \geq 2$.

that is ~~if~~ (x,y) such that $x^2+y^2 \geq 2$.

<u>Range:</u> Range of cosine function is $[-1,1]$.



Graph of $\cos(x)$

~~$\sqrt{x^2+y^2-2}$ can tak~~.

Range of $f(x,y)$ is $[-1,1]$.

Level curves

Given a number z_0.
(x,y) such that $f(x,y)=z_0$ is called a ~~level~~ level curve of f.
(trace at $z=z_0$ of surface $f(x,y)=z$)

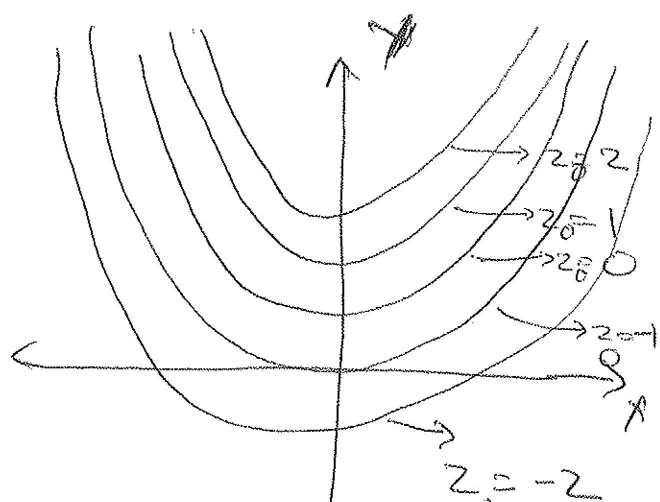
- Pick a number k.
- ~~P~~ In $f(x,y)=z$ replace z by k.
- Plot the curve $f(x,y)=k$.
- Repeat for some other numbers k.

Draw level curves of
$f(x,y) = y-x^2-1$.

- Level curves for $z_0 = 0$.
  $y-x^2-1=0$
  $\Rightarrow x^2=y-1$

Level curve for $z_0=1$
$y-x^2-1=1$
$\Rightarrow x^2=y-2$.

~~Let~~



Closer level curves: ~~chan~~ function changing ~~rapidly~~ slowly.
Wider spaced level curve: function changing ~~slowly~~ rapidly.

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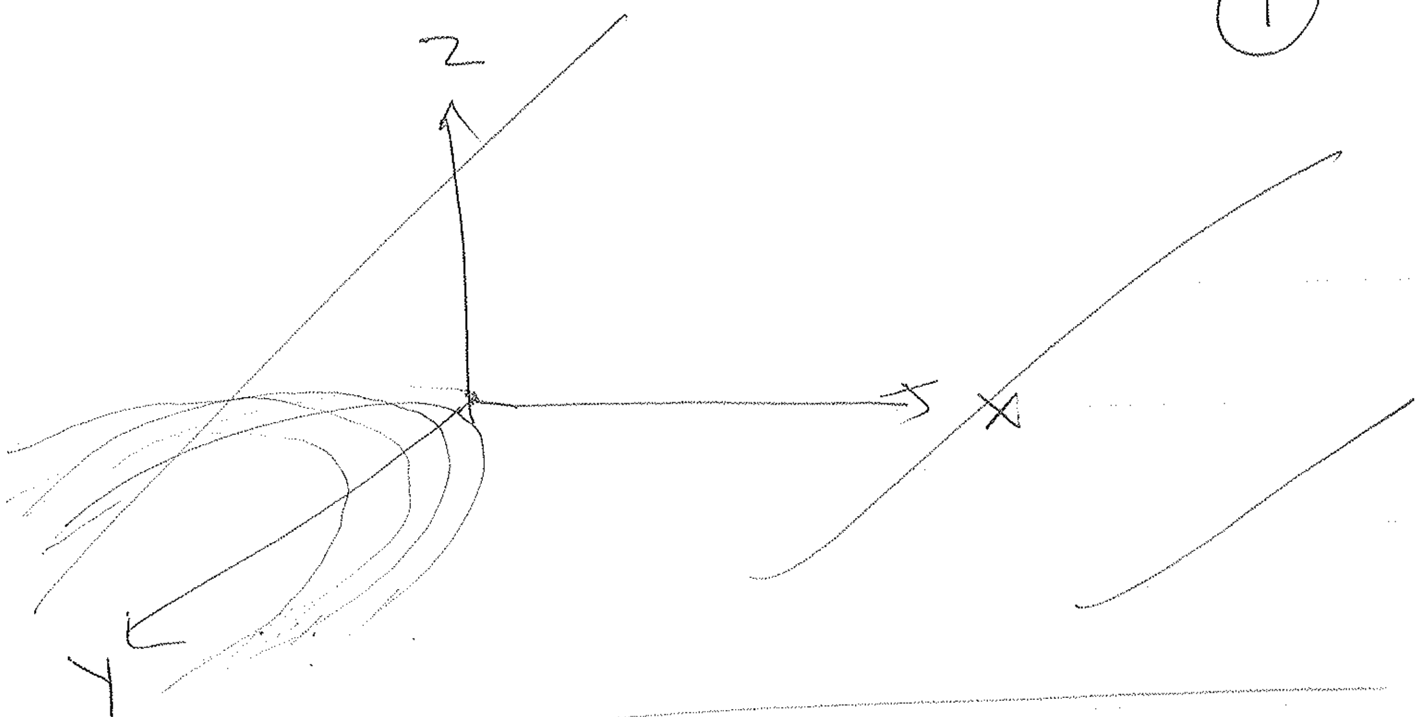


Try level curves of ~~$z = x^2 + y^2$~~ $f(x,y) = \ln(x^2+y^2-3)$

Level curves of $\boxed{z_0 = 0}$

$\ln(x^2+y^2-3) = 0$

$e^{\ln(x^2+y^2-3)} = e^0 = 1$

$\Rightarrow (x^2 + y^2 - 3) = 1$

$\Rightarrow \boxed{x^2 + y^2 = 4}$

$\boxed{z_0 = 1}$

$\Rightarrow \ln(x^2+y^2-3) = 1$

$\Rightarrow e^{\ln(x^2+y^2-3)} = e^1$

$\Rightarrow x^2+y^2-3 = e$

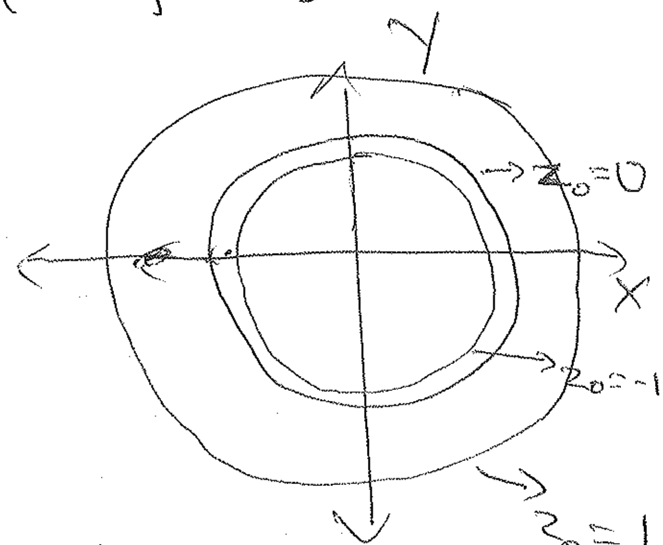
$\Rightarrow \boxed{x^2+y^2 = 3+e}$

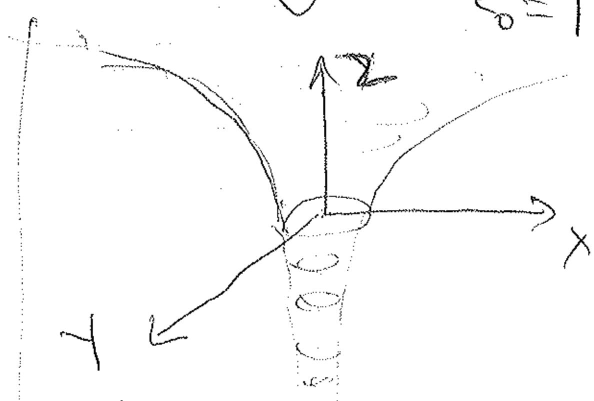
$\boxed{z_0 = -1}$

$\ln(x^2+y^2-3) = -1$

$\Rightarrow x^2+y^2-3 = e^{-1} = \frac{1}{e}$

$\Rightarrow \boxed{x^2+y^2 = 3+\frac{1}{e}}$





graph of $z = \ln(x^2+y^2-3)$

Next ~~cla~~ week: derivatives of functions

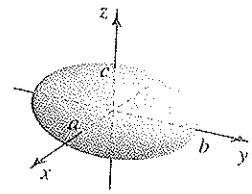
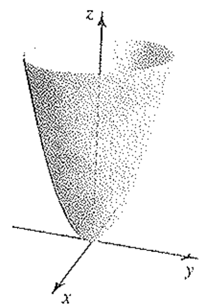
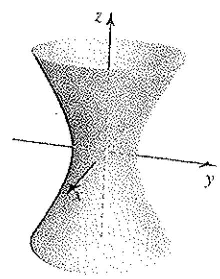
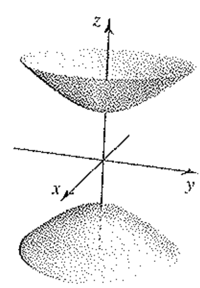
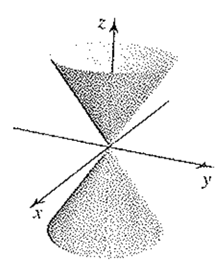
$f(x,y) = x^2 + y^2$

Take derivative with respect to x and y separately. $\boxed{\text{Review derivatives}}$

$f_x(x,y) = 2x$

$f_y(x,y) = 2y$.

Table 12.1

Name	Standard Equation	Features	Graph
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	All traces are ellipses.	
Elliptic paraboloid	$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Traces with $z = z_0 > 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are parabolas.	
Hyperboloid of one sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ are ellipses for all z_0. Traces with $x = x_0$ or $y = y_0$ are hyperbolas.	
Hyperboloid of two sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ with $\lvert z_0 \rvert > \lvert c \rvert$ are ellipses. Traces with $x = x_0$ and $y = y_0$ are hyperbolas.	
Elliptic cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$	Traces with $z = z_0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are hyperbolas or intersecting lines.	
Hyperbolic paraboloid	$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$	Traces with $z = z_0 \neq 0$ are hyperbolas. Traces with $x = x_0$ or $y = y_0$ are parabolas.