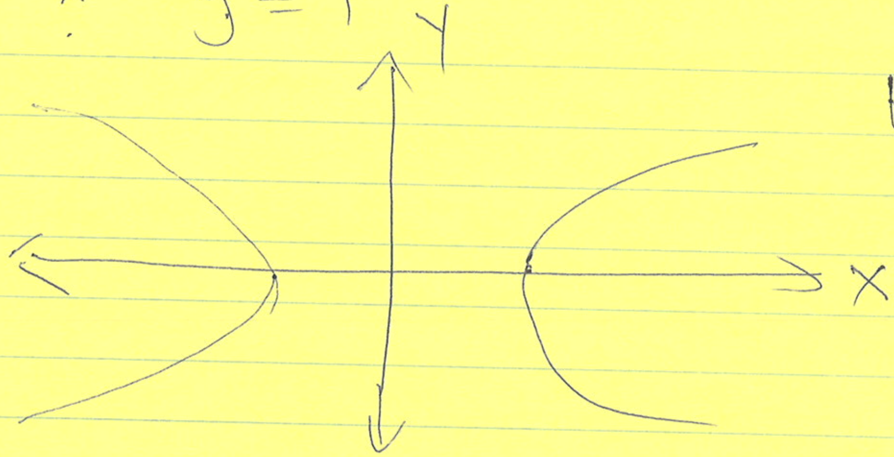
$x^2 - y^2 = 1$

Hyperbola.

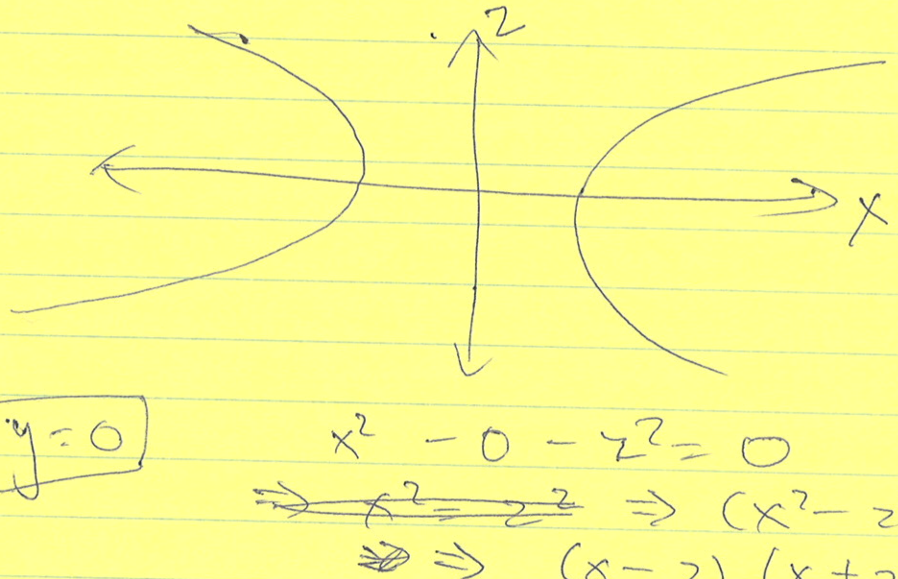


Given the surface $x^2 - y - z^2 = 0$.

Find the trace for $y = 1$ & $y = 0$.

$\boxed{y=1}$ $x^2 - 1 - z^2 = 0$

$\Rightarrow x^2 - z^2 = 1 \longrightarrow$ Hyperbola.

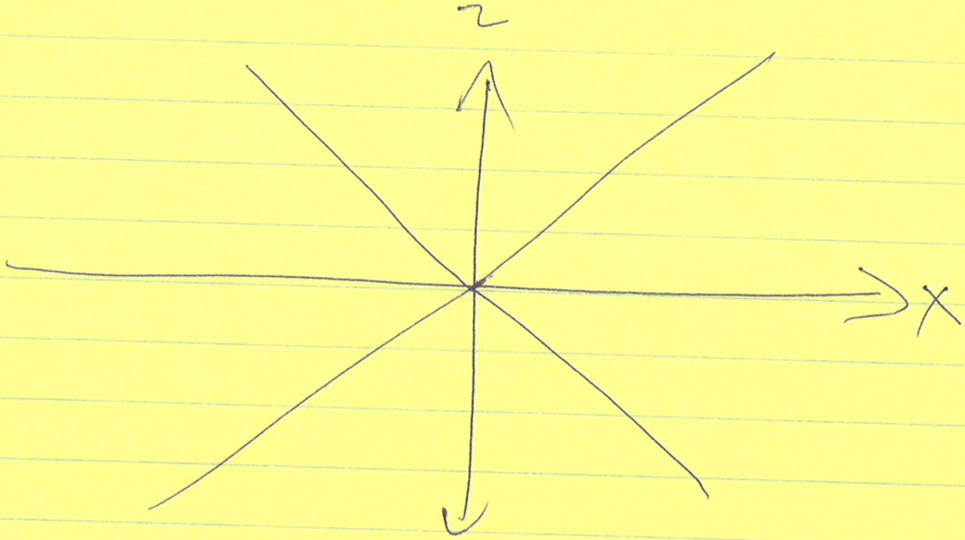


$\boxed{y=0}$ $x^2 - 0 - z^2 = 0$

$\Rightarrow \cancel{x^2 = z^2} \Rightarrow (x^2 - z^2) = 0$

$\Rightarrow (x-z)(x+z) = 0$

$\Rightarrow x - z = 0$ or $x + z = 0$



Level curves: Find level curves for

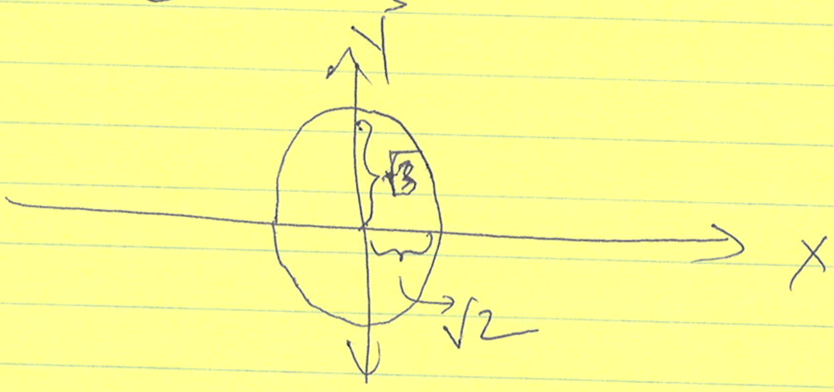
$$f(x,y) = \ln\left(\frac{x^2}{2} + \frac{y^2}{3}\right)$$

In general find the range first

Solution: $\boxed{z = 0}$

$$\ln\left(\frac{x^2}{2} + \frac{y^2}{3}\right) = 0$$

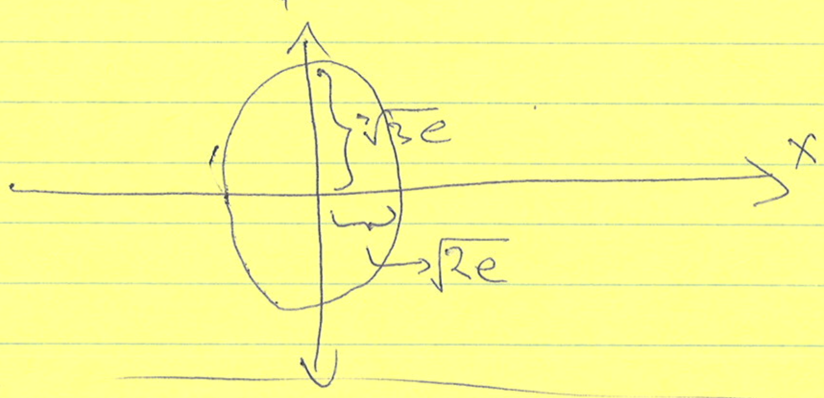
$$\frac{x^2}{2} + \frac{y^2}{3} = e^0 = 1$$



In general do 3 + level curves.

$\boxed{z=1}$ $\quad \ln\left(\frac{x^2}{2}+\frac{y^2}{3}\right)=1$

$$\Rightarrow \frac{x^2}{2}+\frac{y^2}{3}=e^1=e.$$



Find x, y, z intercepts of the plane perpendicular to $\langle 4, 5, 1\rangle$ and passing through $(1,1,1)$.

Solution: The equation is

$$4x + 5y + z = d.$$

~~Plug~~ Plugging $(1,1,1)$ in; we get

$$d = 10$$

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The equation is $4x+5y+z=10$.

x intercept is when $y=0, z=0$

that is x intercept $= \frac{5}{2}$.

Similary y-intercept is 2.

z intercept is 10.

Compute the maximum and minimum of the function x^2+xy+y^2 on $x^2+y^2 \leq 9$.

<u>Solutions</u> For the region $x^2+y^2<9$.

<u>Critical Points:</u> $f(x,y) = x^2+xy+y^2$

$f_x = 2x+y = 0$ — ①

$f_y = 2y+x = 0$ — ②

By ①, we get $2x = -y$.

~~By 2 Plu~~

Plugging it into ②, we get

$$x + 2(-2x) = 0.$$
$$\Rightarrow \quad 3x = 0$$
$$\Rightarrow \quad x = 0$$
$$\Rightarrow \quad y = 0.$$

Minimum,

Thus $(0,0)$ is the only critical point, the value $f(0,0) = 0$

For the boundary ; $x^2 + 9y^2 = 9$.

use Lagrange Multiplier.

$$f(x,y) = x^2 + xy + y^2$$

Constraint

$$g(x,y) = x^2 + 9y^2 - 9.$$

$$f_x = 2x + y \qquad g_x = 2x$$
$$f_y = x + 2y \qquad g_y = \cancel{18y}\ 2y.$$

Equations:

$$2x+y = \lambda 2x \quad \text{--- } (1)$$
$$x+2y = \lambda 2y \quad \text{--- } (2)$$
$$x^2+y^2-9 = 0 \quad \text{--- } (3)$$

By (1) . $\dfrac{2x+y}{2x} = \lambda$ ...

Plugging into (2), we get.

$$x+2y = \frac{(2x+y)}{\cancel{2}x}(\cancel{2}y)$$

$$\Rightarrow x^2 + \cancel{2xy} = \cancel{2xy} + y^2$$

~~Plug~~ Plugging into (3), we get

$$2x^2 - 9 = 0.$$
$$x = \pm\sqrt{\frac{9}{2}} = \pm\frac{3}{\sqrt{2}}.$$

If $x = \frac{3}{\sqrt{2}}$ then $y = \pm\frac{3}{\sqrt{2}}$

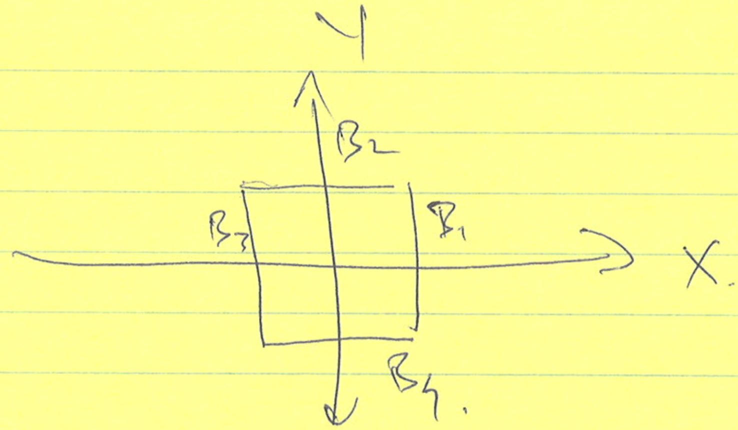
$x = -\frac{3}{\sqrt{2}}$ then $y = \pm\frac{3}{\sqrt{2}}$.

We get 4 points.

Maximum

$$\boxed{f\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right) = f\left(-\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right) = \frac{27}{2}}$$

$$f\left(\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right) = f\left(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right) = \frac{9}{2}.$$



$f(x,y)$
On B_1
$f(1,y)$
$-1 \le y \le 1$

Unbounded Region:

→ there need not exist absolute maxima or minima.

Find f_x, f_y → Find critical points

local Maxima | local minima | Saddle points

Riemann Sums

Represent $\int_1^5 x^2\,dx$ as a limit of a sum.

$a=1$, $b=5$, $f(x)=x^2$

$\Delta_x = \frac{4}{n}$, $x_k = a + k\Delta_x = 1 + k\frac{4}{n} = 1 + \frac{4k}{n}$.

Using left Riemann Sums, we get.

$$\int_1^5 x^2\,dx = \lim_{n\to\infty} \Delta_x \sum_{k=1}^{n} f(x_{k-1})$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{k=1}^{n} \left(1 + \frac{4(k-1)}{n}\right)^2$$

→ Try computing.

→ And working backwards.

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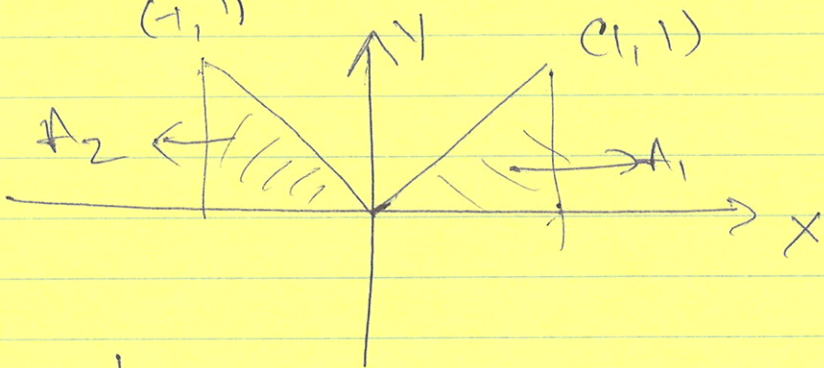
Compute $\int_{-1}^{1} |x| \, dx$

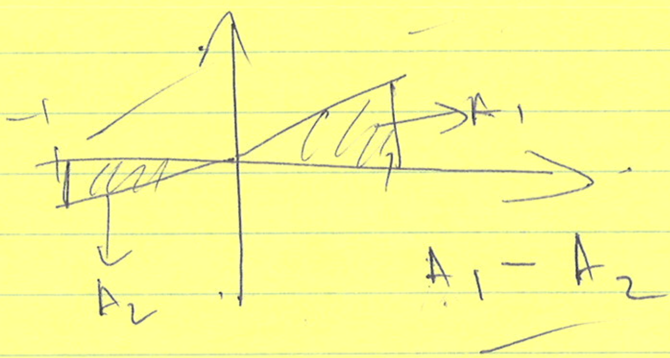
$|x| = x \quad \text{if } x \geq 0$

$|x| = -x \quad \text{if } x \leq 0$



$$\int_{-1}^{1} f(x)\,dx = A_1 + A_2$$

$$= \tfrac{1}{2} 1 \cdot 1 + \tfrac{1}{2} 1 \cdot 1 = 1$$



$A_1 - A_2$