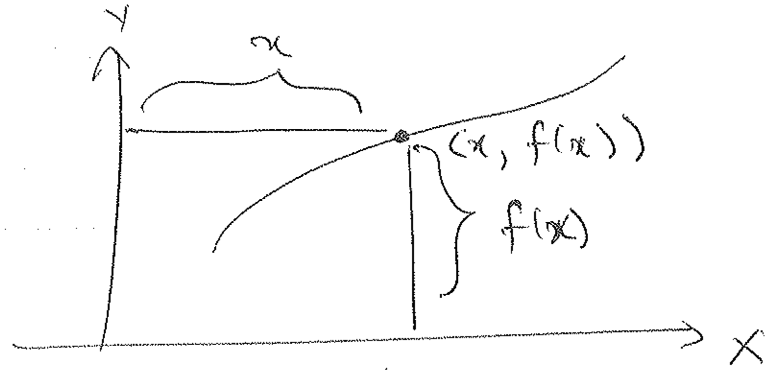
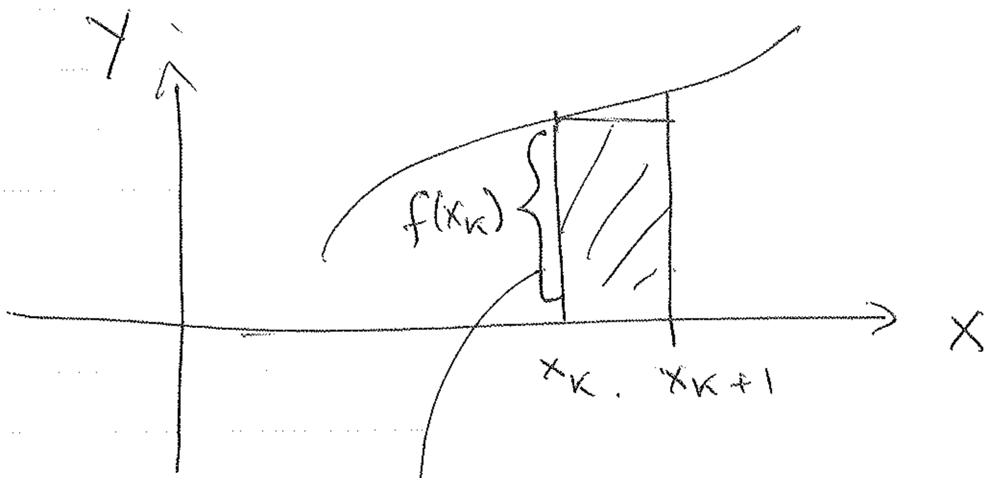
Extra Notes for Riemann Sum

Graph of $f(x)$

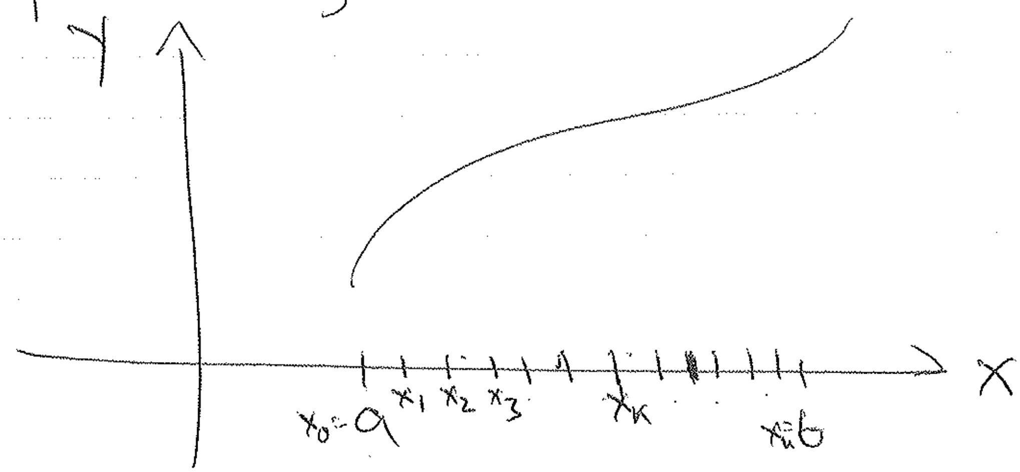




Area: $f(x_k)\underbrace{(x_{k+1} - x_k)}_{\Delta_x}$.

~~Each s_i~~

Divide interval $[a, b]$ into intervals of equal lengths.

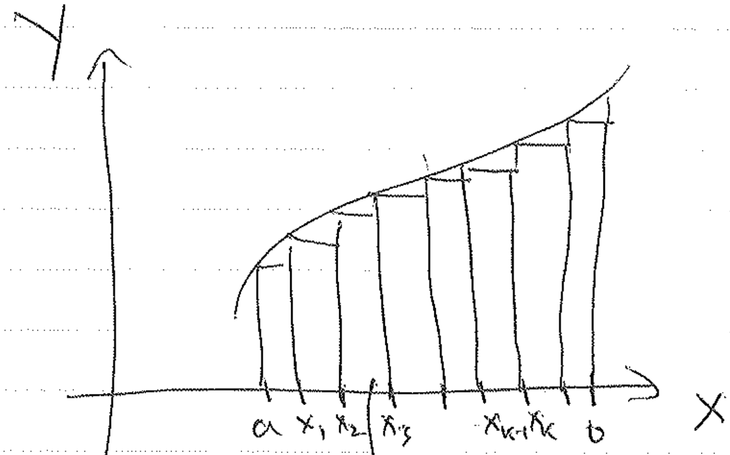


Length of interval is $\left(\frac{b-a}{n}\right) = \Delta_x$

$x_k = a + k(\Delta_x)$ ~~$= a + k$~~

and draw rectangles upto $f(x_k^*)$

Left Riemann Sum

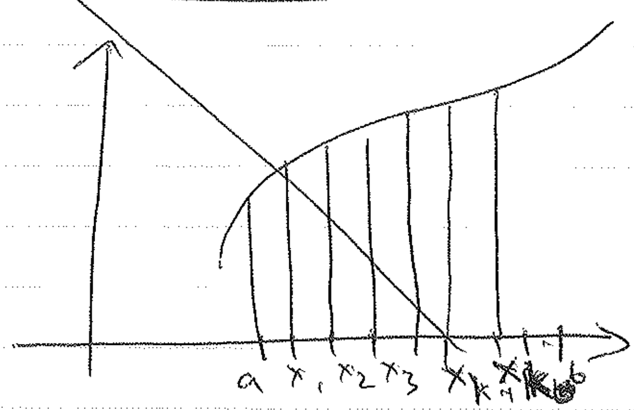


$$\text{Area} = \Delta x \, f(x_0) + \Delta x \, f(x_1) + \cdots + \Delta x \, f(x_k) + \cdots + \Delta x \, f(x_{n-1})$$

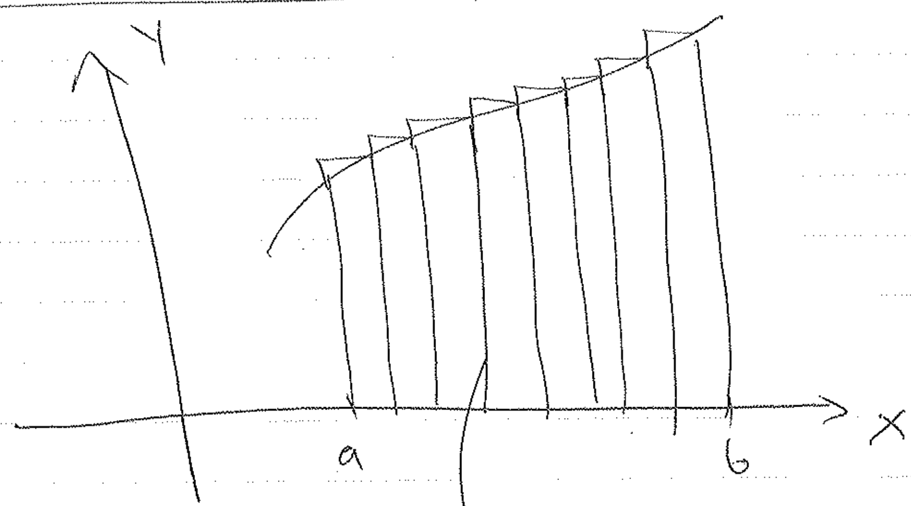
$$= \sum_{k=1}^{n} \Delta x \, f(x_{k-1})$$

$$= \Delta x \sum_{k=1}^{n} f(x_k).$$

Right Riemann Sum



Right Riemann Sum



$$\text{Area} = \Delta_x f(x_1) + \Delta_x f(x_2) + \cdots + \Delta_x f(x_k) + \cdots \cancel{+\Delta_x ff} + \Delta_x f(x_n)$$

$$= \sum_{k=1}^{n} \Delta_x f(x_k).$$

$$= \Delta_x \sum_{k=1}^{n} f(x_k).$$

Similarly for middle Riemann Sum.

__In general:__ Riemann Sum is $\Delta_x \sum_{k=1}^{n} f(x_k^*)$

$$\cancel{\sum_{k=1}^{20} \frac{2}{20(}}$$

Which function and ~~for~~ what interval gives ~~gives~~ Riemann Sum.

$$\sum_{k=1}^{20} \frac{2 \sin\left(\frac{k-1}{20}\right)}{20\left(1+\left(\frac{k-1}{20}\right)^2\right)}$$

~~Compute~~ $\Delta_x \sum_{k=1}^{n} f(x_k^*)$

n and Δ_x $\quad n = 20$

$$\Delta_x = \frac{1}{20} \quad \left(= \frac{b-a}{n}\right)$$

Function and x_k^*

$$\sum_{k=1}^{20} \frac{2 \sin\left(\frac{k-1}{20}\right)}{20\left(1+\left(\frac{k-1}{20}\right)^2\right)} = \frac{1}{20} \sum_{k=1}^{20} \frac{2\sin\left(\frac{k-1}{20}\right)}{\left(1+\left(\frac{k-1}{20}\right)^2\right)}$$

Comparing with $\Delta_x \sum_{k=1}^{20} f(x_k^*)$

tells us $x_k^* = \frac{k-1}{20}$.

Left Riemann Sum: $x_k^* = x_{k-1} = a + (k-1)\Delta_x = a + \frac{(k-1)}{20}$.

Comparing with $x_k^* = \frac{k-1}{20}$.

Find a and b: We arrive at $a = 0$, $b = a + n\Delta_x$
$= 0 + 20\frac{1}{20} = 1$

Function: $f(x_k^*) = f\left(\frac{k-1}{20}\right) = \frac{2 \sin\left(\frac{k-1}{20}\right)}{1+\sin\left(\frac{k-1}{20}\right)}$.

So $f(x) = \dfrac{2\sin x}{1+\sin(x)}$.

This is the ~~Left~~ Left Riemann Sum for

$f(x)$ from 0 to 1 with $n=20$.

partitions.