Math 105 Sample Midterm 1 - Version 2

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Time: 50 minutes   *By* Attempt 7 of the following questions:

1. Consider the planes

A: $2x + 3y - z = 2$
B: $-3x + 2y + 5z = 6$
C: $-6x - 9y + 3z = 7$
D: $3x + 5y + 21z = 0$

In each of the following statements, circle either P (for parallel), O (for orthogonal), or N (for neither parallel nor orthogonal) to make the statement true.

(a): A and B are P / O / N.
(b): A and C are P / O / N.
(c): A and D are P / O / N.
(d): B and C are P / O / N.
(e): B and D are P / O / N.
(f): C and D are P / O / N.

2. Let $h(x, y) = \sqrt{x^2 + y^2 - 6x + 10}$.
(a) Find the range of h.
(b) Find the domain of h.
(c) Find $h_{yy}(1, -2)$.

3. Let $f(x, y) = \ln(x^2 + y^2 - 3)$.
(a) Sketch the level curves $z_0 = f(x, y)$ with $z_0 = 0$, $z_0 = 1$, $z_0 = \ln(6)$.
(b) Where does the graph of f intersect the x-axis?

4. Suppose $f(x, y)$ has continuous partial derivatives of all orders $f_x(0, 0) = f_y(0, 0) = 0$.
(a) If $f_{xx}(0, 0) \leq 0 \leq f_{yy}(0, 0)$ and $f_{xy}(0, 0) \neq 0$, is $(0, 0)$ a local maximum point, a local minimum point, or a saddle point?
(b) If $f_{xy}(0, 0) = 5$, what is the value of $f_{yx}(0, 0)$?

5. Find all critical points of the function $f(x, y) = xy(1 - x - y)$. Classify each point as a local maximum, local minimum, or saddle point.

6. Find the absolute maximum and minimum values of $f(x, y) = 2x^3 - 7x^2 + y^4$ on the closed disk $\{(x, y) : x^2 + y^2 \leq 1\}$.

7. An axis-aligned rectange is a rectangle whose sides are parallel to the coordinate axes. Find the dimensions of the axis-aligned rectangle with maximum area that can be inscribed in the ellipse $x^2 + 4y^2 = 4$. Use Lagrange multipliers to solve this problem. Start by identifying the objective function and the constraint.

8. ~~Find~~ Compute $\int_4^8 |x + 3| \, dx$

9. Use Riemann Sums to compute $\int_1^2 (3x^3 + x) \, dx$.