AN INTRODUCTION TO HOM-SHIFTS

Symbolic dynamics is a multifaceted subject which arises in the conjunction of statistical physics, ergodic theory and computer science. It is the study of the space of configurations (functions from a group G to a finite set A) which is invariant under the natural translation action of G and avoids a given list of forbidden patterns (functions from finite subsets of G to A). Such a space of configurations is called a shift space. Starting with Smale's treatise on differentiable dynamical systems [9] there was an explosion of work on symbolic dynamics when the concerned group was $\mathbb{Z}$. In a very different direction, thermodynamic formalism on one hand [8] and connections with theoretical computer science on the other [7] gave impetus to look at the $\mathbb{Z}^d$ analogues. In this course we will be motivated by the connections with thermodynamic formalism and focus on a special class of shift spaces called hom-shifts. Owing to their inherent symmetry they are more accessible to questions as compared to $\mathbb{Z}^d$ shift spaces in general and at the same time, they arise as important statistical physics models and are rich in their behaviour. We now describe the tentative structure of the course.

Lecture 1: I will motivate the subject in this lecture. We will see how it arises as an analogue of one dimensional dynamics, statistical physics models and gives rise to interesting questions in theoretical computer science. I will end this session with an introduction to entropy and related concepts.

Lecture 2: This will be an exercise session in which we will familiarise ourselves with entropy.

Lecture 3: This will be a student lecture. I will need a volunteer who is familiar with some probability and entropy theory (if not statistical physics models which will be even better).

This lecture intends to give instances of uniqueness and non-uniqueness of measures of maximal entropy and will be based on [10, 3].

Lecture 4: In this lecture we will explore further into measures of maximal entropy and related questions. We will spend some time talking about recent results by Peled and Spinka [6], briefly delve into approximation of entropy and lay down a lot of questions. We will also introduce hom-shifts and rectangular tiling shifts in this lecture.

Lecture 5: This will be a student lecture. There are no specific background requirements for the volunteer here but familiarity with combinatorics and symbolic dynamics will help.

This lecture will focus on mixing properties of hom-shifts and starting with some general background [2] will focus on recent some recent results [1, 4].

Lecture 6: In this lecture we will focus on the universality of hom-shifts [5].

References

[1] N. Alon, R. Briceño, N. Chandgotia, A. Magazinov, and Y. Spinka. Mixing properties of colorings of the $\mathbb{Z}^d$ lattice. https://arxiv.org/abs/1903.11685, 2019.

[2] M. Boyle, R. Pavlov, and M. Schraudner. Multidimensional sofic shifts without separation and their factors. *Trans. Amer. Math. Soc.*, 362(9):4617–4653, 2010.

[3] R. Burton and J. E. Steif. Non-uniqueness of measures of maximal entropy for subshifts of finite type. *Ergodic Theory Dynam. Systems*, 14(2):213–235, 1994.

[4] N. Chandgotia and B. Marcus. Mixing properties for hom-shifts and the distance between walks on associated graphs. *Pacific J. Math.*, 294(1):41–69, 2018.

[5] N. Chandgotia and T. Meyerovtich. Borel subsystems and ergodic universality for compact $\mathbb{Z}^d$-systems via specification and beyond. https://arxiv.org/abs/1903.05716, 2019.

[6] R. Peled and Y. Spinka. Rigidity of proper colorings of. *arXiv preprint arXiv:1808.03597*, 2018.

[7] R. M. Robinson. Undecidability and nonperiodicity for tilings of the plane. *Invent. Math.*, 12:177–209, 1971.

[8] D. Ruelle. *Thermodynamic formalism*. Cambridge Mathematical Library. Cambridge University Press, Cambridge, second edition, 2004. The mathematical structures of equilibrium statistical mechanics.

[9] S. Smale. Differentiable dynamical systems. *Bull. Amer. Math. Soc.*, 73:747–817, 1967.

[10] J. van den Berg and J. E. Steif. Percolation and the hard-core lattice gas model. *Stochastic Process. Appl.*, 49(2):179–197, 1994.